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Product availability in competitive and cooperative dual-channel distribution with stock-out based substitution

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A B S T R A C T

This paper investigates the impact of customers' stock-out based substitution on the product availability and the channel efficiency of a dual-channel supply chain, which consists of a supplier distributing a single product to customers through both its wholly owned direct channel and an independent retailer. The supplier and its retailer, with the objective of optimizing their own profit, simultaneously choose their own base-stock level to satisfy the stochastic demand from the customers whose channel preferences are heterogeneous and may be affected by each channel's product availability. The customers dynamically substitute between the two channels in the event of a stock-out. The result shows that the effect of the stock-out based substitution may increase or decrease the efficiency of a decentralized supply chain. It is found that while the integrated supplier–retailer may consolidate the base-stock levels to benefit from stock-out based substitution, the independent supplier and retailer are more inattentive to customers' stock-out based substitution. Thus, the competitive base-stock levels of the decentralized dual-channel supply chain rarely agree with the system optimal levels. Various contracts are examined to shed light on channel coordination mechanisms. In addition, it is shown that the channel efficiency of the dual-channel distribution can be improved by the emergence of Stackelberg leadership from either the supplier or the retailer.

1. Introduction

Facilitated by the advent of e-commerce and the rapid development of third-party logistics, a progressive number of suppliers have ventured to engage in direct sales by introducing an Internet based channel alongside the incumbent retail channel. When a supplier sells both through a retailer and directly to consumers, such a distribution channel is called dual-channel (or multi-channel) distribution. In a dual-channel distribution setting, a supplier and its retailer sell essentially the same product. Therefore, customers have alternatives to choose the channel that is better suited to their needs. They may alternatively switch to the other channel when a stock-out occurs in their preferred channel.

Stock-out based substitution is a common consumer behavior. A recent survey in the *Harvard Business Review* (Corsten and Gruen, 2004) reveals that, depending on the product category, 21–43% of consumers faced with a stock-out will actually go to another store to buy the item. Practitioners have suggested that reducing customers' search cost by adopting new technologies which let customers access real-time inventory information may stimulate stock-out based substitution and thus improve channel efficiency.

> "For customers, having the freedom to shop at their favorite store online is great, but the shopper may not know that a sweater she wants that is out of stock on the Web could be hanging on the rack at the nearest local store. There is a disconnect between online and offline channels, which for retailers means store associates cannot keep track of inventory across all channels or fulfill a request for an item that their store does not have in stock. Ultimately, this could translate into lost revenues..." (Lawson, 2001)

Could increasing stock-out based substitution really enhance channel efficiency in a dual-channel distribution? Unfortunately, the answer to this question is not immediately clear. On one hand, stock-out based substitution may reduce the amount of lost sales since demand at one channel that is out of stock may be captured by the other channel with available inventory. On the other hand, however,

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this sort of stock-out based substitution in a dual-channel supply chain may create strategic interactions between a supplier and its intermediary with respect to their stocking decisions. The resulting conflict can possibly deteriorate channel efficiency when the supplier-owned direct channel establishes the supplier as a direct competitor to its retailer.

Interestingly, the type of channel conflict in a dual-channel distribution arises from a rather unique competitive situation: a supplier and its retailer are engaged in both vertical and horizontal competition simultaneously (see Fig. 1). Specifically, vertical competition is associated with inefficient price double marginalization (Spengler, 1950), which occurs when the supplier, as a result of selling at a wholesale price above its marginal cost to an intermediary, induces its retailer to set a retail price above what it would be if it faced the true marginal cost of the channel. Double marginalization is a well-known example of supply chain inefficiency caused by vertical competition. When the retail price is fixed, the existence of double marginalization may cause the retailer to carry too little inventory relative to the optimal amount in a one-period model where a manufacturer sells to a retailer facing uncertain demands (Pasternack, 1985; Lariviere and Porteus, 2001). Also, in a multiple-period model where each firm uses a base-stock policy, a decentralized supply chain generally understocks due to double marginalization (Cachon and Zipkin, 1999; Cachon, 2001; Axsäter, 2001; Caldentey and Wein, 2003).

On the other hand, horizontal competition occurs when firms sell substitutable products and competitively stock the products. Research studies on horizontal inventory competition are established in both operations and marketing literature (e.g., Parlar and Goyal, 1984; Balachander and Farquhar, 1994; Lippman and McCardle, 1997; Mahajan and van Ryzin, 2001; Netessine and Rudi, 2003). In general, it is found that while there might be cases where it results in product understocking, horizontal competition results in overstocking for most practical situations, causing a loss in profit relative to the monopoly case.

What happens when both types of competition co-exist in a supply chain? In addition to the literature analyzing the impact of pure-vertical and pure-horizontal competition on supply chain performance, there is also a recent stream of literature on the effect of combined vertical–horizontal competition (e.g., Anupindi and Bassok, 1999; Van Ryzin and Mahajan, 2000; Netessine and Zhang, 2005). Loosely speaking, when both double marginalization and substitutability co-exist, the inefficient understocking due to vertical competition in a supply chain can be counteracted by the overstocking due to horizontal competition; that is, the combined effect of both vertical and horizontal competition may help to improve supply chain performance. This result, which relies on symmetry assumption across distribution channels, may not always be applicable in a multi-channel distribution where asymmetry enters the picture. Moreover, due to a single-period setting, the studies in this stream of literature generally assume that the supplier(s) carries no stock at the upper echelon. Therefore, since inventory competition is only among firms located within the same supply chain echelon, these studies are unable to provide useful guidance for managing a multi-channel distribution in which inventory competition is among firms located both within the same echelon and in different echelons.

Thus far, due likely to the analytical intricacies of the problem, there is no study attempting to address the stocking behaviors under both horizontal and vertical inventory competition with a multi-echelon multi-channel framework. The objective of this paper is to enhance our understanding of this essential subject. Specifically, motivated by the modeling techniques in Chiang and Monahan (2005), who investigate a two echelon dual-channel inventory problem for an integrated distribution channel, this study, from an opposite slant, constructs an inventory game for a decentralized supply chain wherein a supplier uses both its wholly owned direct channel and a third-party retailer to distribute its products to customers with heterogeneous channel preference. The two channel members independently choose their respective stocking level to satisfy the stochastic demand from the customers who may dynamically substitute between the two sales channels in the event of a stock-out. We investigate whether the supplier and its retailer with competition are likely to carry over more or less stocks than they would without competition. Based upon the findings on the channel members' stocking behaviors, we further examine numerous coordination mechanisms for improving channel efficiency. Since Chiang and Monahan (2005) as well as other related studies, such as Cattani and Souza (2002), have not addressed the issues concerning inventory competition and coordination, our analysis yields distinct and pertinent insights for those multi-channel companies operating with decentralized supply chains. A stylized example of such companies is Sony, which makes its HDTV products available for sale at Best Buy, a third-party retailer, while selling the identical products on its owned online channel. It has been observed that, the prices for most HDTV products on Sony's online channel exactly match those at Best Buy (Cattani et al., 2006). Although such a price matching strategy may help to alleviate channel conflict caused by price competition, as revealed in this study, the independent stocking behaviors of the two companies may still result in channel inefficiency. Apparently, how to effectively manage supply chain inventories with multiple competing channels is essentially a challenging subject for Sony and many other companies with a similar channel context, such as Xerox, Holmes, Gateway, and Ethan Allen.

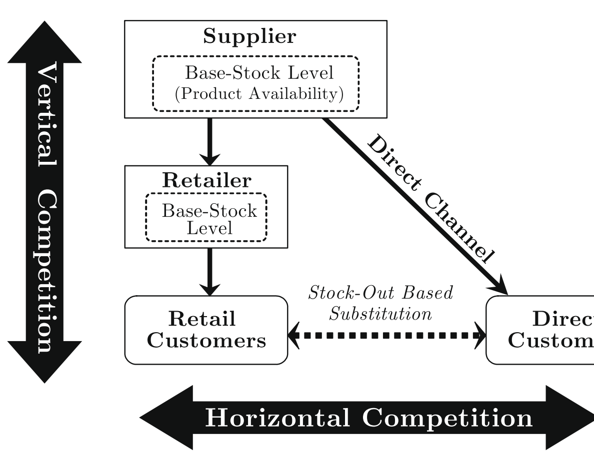


Fig. 1. Competitive dual-channel supply chain.

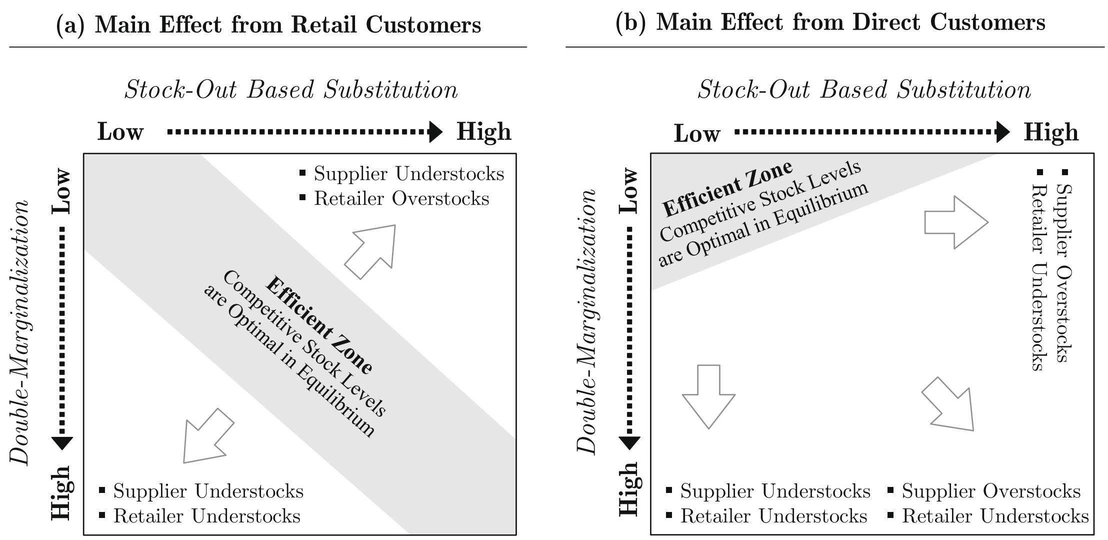


Fig. 2. Competitive product availability in a dual-channel distribution.

1.1. Preview of the main results

The effect of combined vertical–horizontal competition in a dual-channel distribution is intuitively ambiguous, especially when customers' stock-out based substitution is asymmetrical, i.e., when the proportion of retail customers who will search and switch to the other channel is different from that of direct customers when a stock-out occurs.[1] The result of our analysis indicates that depending on the type of asymmetry of customers' stock-out based substitution, the combined effect of vertical–horizontal competition may increase or decrease the system efficiency in a dual-channel supply chain. In particular, if the stock-out based substitution of retail customers is more considerable than that of direct customers, horizontal competition may be beneficial to a dual-channel distribution system, as it helps to improve channel efficiency by counterbalancing the understocking behavior of both the supplier and the retailer caused by a high degree of double marginalization (see Fig. 2a). If it is the opposite, the effect of intensive vertical–horizontal competition generally exacerbates the channel inefficiency. The reasoning is that the retailer's understocking behavior caused by the effect of vertical competition is further aggravated by the effect of horizontal competition, which, in turn, compels the supplier to overstock (see Fig. 2b).

Our result reveals that while the integrated supplier–retailer may reduce the base-stock levels to benefit from stock-out based substitution through the risk pooling effect, the independent supplier and retailer are more inattentive to customers' stock-out based substitution. As a result, the competitive base-stock levels of the decentralized dual-channel supply chain rarely agree with the system optimal base-stock levels; that is, the decentralized dual-channel supply chain is inefficient in most cases. Can the incentives of the channel entities in an inefficient dual-channel supply chain be aligned through a contract such that the channel performs at the optimal level in equilibrium? We find that although a contract that guarantees the channel coordination in any scenario can be identified, such a contract is not straightforward to implement. Nevertheless, there are several contracts in the supply chain literature that are relatively unsophisticated and may be used as alternatives to mitigate the implementation hassle if they are applied in the appropriate situations.

This paper assumes that both channel members are blind to how their own decision on a product availability affects their partner's decision, and thus channel leadership does not exist in the marketing channel. Our further investigation of two Stackelberg Leader–Follower games show that the efficiency of the dual-channel distribution can be improved by the emergence of Stackelberg leadership from either the supplier or the retailer.

1.2. Other related literature

The implications of a stock-out are serious to all members of the marketing channel (Schary and Christopher, 1979). Given the importance, several research studies have examined the relationship between the stock-out and the value of the potentially resulting lost sales by investigating consumer response to stock-outs (e.g., Walter and Grabner, 1975; Motes and Castleberry, 1985; Anupindi et al., 1998; Fitzsimons, 2000; Zinn and Liu, 2001). In addition, from a strategic viewpoint, a number of marketing researchers have modeled the tactical interactions in a distribution channel based on consumer response to stock-outs (e.g., Hess and Gerstner, 1987, 1990, 1998; Balachander and Farquhar, 1994; Wilkie et al., 1998). While their studies provide various insights and competitive rationales regarding product availability of the marketing channel, the results are generally not applicable to the dual-channel distribution context of this paper.

Multi-echelon inventory control problem, first introduced by Clark and Scarf (1960), is known to be a challenging research area. Due the complexity and intractability of the problem, the adoption of single location, single echelon models for the inventory systems is recommend by Hadley and Whitin (1963). Sherbrooke (1968) constructed the METRIC model, which is the first multi-echelon inventory model for managing the inventory of service parts. Thereafter, a large set of models seeking to identify optimal lot sizes and base stock levels in a multi-echelon framework have been developed (e.g., Svoronos and Zipkin, 1988; Axsäter, 1990; Dada, 1992; Nahmias and Smith, 1994). In

[1] Many manufactures/suppliers have established regional warehouses to fulfill the demands from local retailers and/or direct customers, which increase the direct customers' likelihood to experience a stock–stock since the average stock level at a regional warehouse is typically lower than that at a firm's central warehouse.

general, this body of literature focuses on centralized decision making in hierarchical inventory systems, ignoring the independence of each supply chain member.

Research on dual-channel distribution in the setting where the upstream echelon is both a supplier to and a competitor of the downstream echelon has emerged only recently. A number of papers in this stream of literature (e.g., Rhee and Park, 2000; Kumar and Ruan, 2006; Chiang et al., 2003; Tsay and Agrawal, 2004b) focus on channel competition and coordination issues. They address strategic price and/or service interactions between upstream and downstream without considering the inventory competition problem that we examine in this paper. Motivated by Anupindi and Bassok (1999) and Van Ryzin and Mahajan (2000), Boyaci (2005) models multi-channel competition for a single-period inventory problem. This model assumes that the manufacturer's does not hold stock at the upper echelon, and thus inventory competition occurs only horizontally. Clearly, such setting is very different from our multi-period framework which involves both vertical and horizontal inventory competition. There are several other papers (e.g., Peleg and Lee, 2002; Bell et al., 2003; Yue and Liu, 2006; Kurata et al., 2007; Dumrongsiri et al., 2008; Mahar et al., 2009) that also address related issues on multi-channel supply chain. However, their foci are different. We refer the readers to Tsay and Agrawal (2004a) for a detailed review of this stream of literature.

The remainder of the paper is organized as follows. Section 2 defines the continuous time Markov chains of the product availability process in the two-echelon dual-channel distribution. Accordingly, the models that are used to measure the performance of centralized and decentralized supply chains are formulated in Section 3. The numerical study of competitive effect is then presented in Section 4. Section 5 contains the analysis of coordination contracts for the dual-channel supply chain. The Stackelberg Leader–Follower games are examined in Section 6, and the final section concludes the paper.

2. Product availability process

Consider a product selling at both the retailer and the supplier's direct channel. The market consists of two types of customers: those who prefer the retail channel (retail customers) and those who prefer the direct channel (direct customers). Demand of the retail customers is satisfied with the on-hand inventory at the retailer, while demand of the direct customers is fulfilled through direct delivery with the on-hand inventory at the supplier. Suppose that the product demand is characterized by a Poisson process $\{N(t), t \geqslant 0\}$ having rate λ. Denoting the proportion of direct customers by α, we first obtain the following lemma, which is essential for establishing the underlying Markov model of this study.

Lemma 1. *(i) The arrivals of direct and retail customers are both Poisson processes having respective rates $\lambda_d = \alpha\lambda$ and $\lambda_r = (1-\alpha)\lambda$. (ii) The two arrival processes are independent.*

The proof of Lemma 1 is given in Appendix A. Note that Cattani and Souza (2002) and Chiang and Monahan (2005) also assume Poisson demand in a similar context. However, they simply consider two Poisson processes without justifying their independency.

Assume that both the supplier and the retailer incur no fixed ordering costs and they both implement one-for-one ordering policies to replenish their inventories. Under such replenishment policies, the inventory positions are kept constant at base-stock levels, and an order for one unit is placed each time a demand arrival is served from stock on hand. The respective base-stock levels for the supplier and the retailer are denoted by S_s and S_r.

When a stock-out occurs in the retailer, a proportion, say β_r, of retail customers are willing to switch to the supplier's direct channel. On the other hand, a proportion β_d of direct customers who incur a stock-out at the supplier are willing to switch to the retailer. In case of a stock-out, customers who are unwilling to shift to the other channel result in lost sales. In addition, customers are lost when both the supplier and the retailer are out of stock simultaneously. The replenishment lead times for the supplier and the retailer are assumed to be independent exponential random variables with means $1/\mu_s$ and $1/\mu_r$, respectively, and the replenishment backorders from the retailer are allowed. A customer served from stock on hand will trigger a replenishment order immediately. Therefore, the information lead time is assumed to be zero.

Let x be the on-hand stock level at the supplier, and y be the on-hand stock level at the retailer. Following a common approach in the multi-echelon inventory literature (e.g. Dada, 1992), the Markov model that captures the model assumptions can be constructed with each state $(x,y) \in \Omega$, where Ω is the state space

$$\Omega = \{(x,y) \in \mathbf{Z}^2 | -S_r \leqslant x \leqslant S_s, 0 \leqslant y \leqslant S_r\}. \tag{1}$$

The stock on hand at the supplier can be negative because we assume that replenishment backorders from the retailer are allowed at the supplier. With the base-stock levels S_s and S_r, the total number of states is $(S_s + S_r + 1)(S_r + 1)$. For each state (x,y), there are four possible events, characterized below, leading to a transition out of the state:

Event $\langle 1 \rangle$: a customer served from stock on hand by the supplier: $(x,y) \to (x-1,y)$;
Event $\langle 2 \rangle$: a customer served from stock on hand by the retailer: $(x,y) \to (x-1,y-1)$;
Event $\langle 3 \rangle$: a replenishment order arrives at the supplier: $(x,y) \to (x+1,y)$;
Event $\langle 4 \rangle$: a replenishment order arrives at the retailer: $(x,y) \to (x,y+1)$.

Fig. 3 illustrates the transition diagram of the Markov model. Let $P_{(x,y)}^{(S_s,S_r)}$ denote the steady-state probability of state (x,y) when the base-stock levels S_s and S_r are specified; $P_{(x,y)}^{(S_s,S_r)} = 0$ if $(x,y) \notin \Omega$. Then the flow balance equations, which require that the total flow out of a state is equal to the total flow into the state for all states, are given by

$$\sum_{k=1}^{4} \eta_{(x,y)}^{\langle k \rangle} P_{(x,y)}^{(S_s,S_r)} = \eta_{(x+1,y)}^{\langle 1 \rangle} P_{(x+1,y)}^{(S_s,S_r)} + \eta_{(x+1,y+1)}^{\langle 2 \rangle} P_{(x+1,y+1)}^{(S_s,S_r)} + \eta_{(x-1,y)}^{\langle 3 \rangle} P_{(x-1,y)}^{(S_s,S_r)} + \eta_{(x,y-1)}^{\langle 4 \rangle} P_{(x,y-1)}^{(S_s,S_r)}, \quad \forall (x,y) \in \Omega, \tag{2}$$

where $\eta_{(x,y)}^{\langle k \rangle}$ is the transition rate from state (x,y) for event k. The left-hand side of (2) reflects the average transitions out of state (x,y), while conversely, the right-hand side of (2) represents the average transitions into state (x,y). For each event k, $k = 1,2,3,4$, the transition rates vary

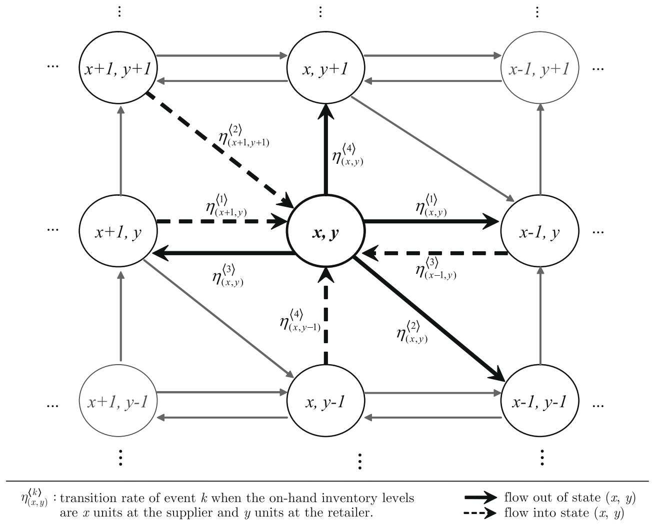


Fig. 3. The transition diagram of the Markov model.

from state to state. In order to find steady-state probabilities, the specific transition rates $\eta^{\langle k\rangle}_{(x,y)}$ in the balance equations need to be precisely encapsulated.

We start with $\eta^{\langle 1\rangle}_{(x,y)}$ and $\eta^{\langle 2\rangle}_{(x,y)}$, the transition rates caused by receiving demands. Based on the model assumptions stated previously, the two respective transition rates as a result of satisfying demand by stock on hand at the supplier and the retailer can be modeled as

$$\eta^{\langle 1\rangle}_{(x,y)} = \phi_{(x)}(\lambda_d + (1-\phi_{(y)})\beta_r\lambda_r), \tag{3}$$

$$\eta^{\langle 2\rangle}_{(x,y)} = \phi_{(y)}(\lambda_r + (1-\phi_{(x)})\beta_d\lambda_d), \tag{4}$$

where $\phi_{(z)}$ is used to detect whether a stock-out occurs or not. Specifically,

$$\phi_{(z)} = \begin{cases} 1 & \text{if } z > 0, \\ 0 & \text{otherwise}. \end{cases} \tag{5}$$

Note that when both channels have stock available ($\phi_{(x)} = 1, \phi_{(y)} = 1$), the total transition rate from state (x,y) due to receiving demand is $\eta^{\langle 1\rangle}_{(x,y)} + \eta^{\langle 2\rangle}_{(x,y)} = \lambda_r + \lambda_d = \lambda$. On the other hand, when both channels are out of stock simultaneously ($\phi_{(x)} = 0, \phi_{(y)} = 0$), this total transition rate is zero. If the stock is out at the retailer ($\phi_{(y)} = 0$) but is available at the supplier ($\phi_{(x)} = 1$), then $\eta^{\langle 1\rangle}_{(x,y)} + \eta^{\langle 2\rangle}_{(x,y)} = \beta_r\lambda_r + \lambda_d$ (a proportion of the retail customers, β_r, will switch to the direct channel). Likewise, the total transition rate due to receiving demand is $\lambda_r + \beta_d\lambda_d$ when the stock is out at the supplier but is available at the retailer.

The rates at which in-transit replenishment orders arrive at the supplier and the retailer are directly affected by the replenishment units delayed at the supplier and the in-transit orders from the supplier to the retailer. Specifically, assuming that the supplier's source has infinite capacity, and the supplier ships a retailer's order immediately provided that inventory is available (i.e., $x > 0$), the two transition rates due to receiving replenishment orders at the supplier and the retailer, respectively, can be written as

$$\eta^{\langle 3\rangle}_{(x,y)} = (S_s - x)\mu_s, \tag{6}$$

$$\eta^{\langle 4\rangle}_{(x,y)} = [S_r - y - [x]^-]^+\mu_r. \tag{7}$$

Math notation follows: $[z]^+ = \max\{0,z\}$, and $[z]^- = \max\{0,-z\}$.

With the transition rates $\eta^{\langle k\rangle}_{(x,y)}$ identified in Eqs. (3), (4), (6), and (7), the balance equations to find steady-state probabilities are below:

$$\begin{aligned}&[\phi_{(x)}(\lambda_d + (1-\phi_{(y)})\beta_r\lambda_r) + \phi_{(y)}(\lambda_r + (1-\phi_{(x)})\beta_d\lambda_d) + (S_s - x)\mu_s + [S_r - y - [x]^-]^+\mu_r]P^{(S_s,S_r)}_{(x,y)} \\ &\quad = \phi_{(x+1)}(\lambda_d + (1-\phi_{(y)})\beta_r\lambda_r)P^{(S_s,S_r)}_{(x+1,y)} + \phi_{(y+1)}(\lambda_r + (1-\phi_{(x+1)})\beta_d\lambda_d)P^{(S_s,S_r)}_{(x+1,y+1)} \\ &\qquad + (S_s - (x-1))\mu_s P^{(S_s,S_r)}_{(x-1,y)} + [S_r - (y-1) - [x]^-]^+\mu_r P^{(S_s,S_r)}_{(x,y-1)}, \quad \forall (x,y) \in \Omega.\end{aligned} \tag{8}$$

For any given set of base-stock levels (S_s,S_r), the subsequent steady-state probabilities are uniquely determined and can be found by solving the following system of linear equations:

$$\mathbf{A}^{(S_s,S_r)}\mathbf{P}^{(S_s,S_r)} = \mathbf{0}, \tag{9}$$

$$\sum_{(x,y)\in\Omega} P^{(S_s,S_r)}_{(x,y)} = 1, \tag{10}$$

where $\mathbf{A}^{(S_s,S_r)}$ is the transition rate matrix, $\mathbf{P}^{(S_s,S_r)}$ is the vector of steady-state probabilities, and Eq. (10) is the normalizing constraint. Note that $\mathbf{A}^{(S_s,S_r)}$ is an $(S_s + S_r + 1)(S_r + 1) \times (S_s + S_r + 1)(S_r + 1)$ matrix and $\mathbf{P}^{(S_s,S_r)}$ is an $(S_s + S_r + 1)(S_r + 1)$-vector. When the base-stock levels $(S_s,S_r) = (2,1)$, for example, the corresponding system of balance equations $\mathbf{A}^{(2,1)}\mathbf{P}^{(2,1)} = 0$ is given by

$$\begin{pmatrix} \lambda_r+\lambda_d & -\mu_s & & & -\mu_r & & & \\ -\lambda_d & \lambda_r+\lambda_d+\mu_s & -2\mu_s & & & -\mu_r & & \\ & -\lambda_d & \lambda_r+\beta_d\lambda_d+2\mu_s & -3\mu_s & & & -\mu_r & \\ & & & 3\mu_s & & & & \\ & & & & \lambda_d+\beta_r\lambda_r+\mu_r & \mu_s & & \\ -\lambda_r & & & & -\lambda_d-\beta_r\lambda_r & \lambda_d+\beta_r\lambda_r+\mu_r+\mu_s & -2\mu_s & \\ & -\lambda_r & & & & -\lambda_d-\beta_r\lambda_r & \mu_r+2\mu_s & -3\mu_s \\ & & -\lambda_r-\beta_d\lambda_d & & & & & 3\mu_s \end{pmatrix} \begin{pmatrix} P_{(2,1)} \\ P_{(1,1)} \\ P_{(0,1)} \\ P_{(-1,1)} \\ P_{(2,0)} \\ P_{(1,0)} \\ P_{(0,0)} \\ P_{(-1,0)} \end{pmatrix} = 0.$$

In the section that follows, the steady-state probabilities, which are crucial for our investigation of the system, will be used to model several measures of channel performance.

3. Product availability and channel efficiency

Suppose that the supply chain operates over an infinite horizon. The supplier sells the product to the retailer at a per unit wholesale price w and to customers at a fixed marginal price d through its own direct channel. The costs incurred by the supplier for each unit of the product sold through the retail channel and the direct channel are c_r and c_d, respectively. Note that c_r and c_d may include the unit production cost and the logistics cost of delivering the product to end customers. In addition, the supplier incurs a per unit inventory holding cost at rate h_s. The retailer purchases the product from the supplier and sells the product to customers at a fixed marginal retail price r. The inventory holding cost per item per time unit at the retailer is h_r. To avoid trivial problems, we assume $c_d \leqslant d$ and $c_r \leqslant w \leqslant r$. Let m_d and m_r be the respective margins of direct and retail sales, $m_d = d - c_d$ and $m_r = r - c_r$. Then, with the steady-state probabilities, we can represent the steady-state expected direct and retail sales volumes as

$$Q_d^{(S_s,S_r)} = \lambda_d \sum_{x=1}^{S_s} \sum_{y=0}^{S_r} P_{(x,y)}^{(S_s,S_r)} + \beta_r\lambda_r \sum_{x=1}^{S_s} P_{(x,0)}^{(S_s,S_r)}, \tag{11}$$

$$Q_r^{(S_s,S_r)} = \lambda_r \sum_{x=-S_r}^{S_s} \sum_{y=1}^{S_r} P_{(x,y)}^{(S_s,S_r)} + \beta_d\lambda_d \sum_{x=-S_r}^{0} \sum_{y=1}^{S_r} P_{(x,y)}^{(S_s,S_r)}. \tag{12}$$

Also, the steady-state average inventories for the supplier and the retailer, respectively, depend on the steady-state probabilities in the following way:

$$I_s^{(S_s,S_r)} = \sum_{x=1}^{S_s} \sum_{y=0}^{S_r} x P_{(x,y)}^{(S_s,S_r)} \quad \text{and} \quad I_r^{(S_s,S_r)} = \sum_{x=-S_r}^{S_s} \sum_{y=1}^{S_r} y P_{(x,y)}^{(S_s,S_r)}. \tag{13}$$

3.1. The vertically integrated (centralized) channel

Assume that all the prices are competitively determined and all the cost-related parameters are exogenous. Then the only decision variables in the system are the base-stock levels. The steady-state expected profit for the whole supply chain can be modeled as:

$$\pi(S_s,S_r) = m_d Q_d^{(S_s,S_r)} + m_r Q_r^{(S_s,S_r)} - h_s I_s^{(S_s,S_r)} - h_r I_r^{(S_s,S_r)}. \tag{14}$$

When the supplier and the retailer are coordinated, the objective is to find base-stock levels that maximize the total supply chain profit specified in (14).

Lemma 2. *There exist stock levels, $\overline{S}_s$ and $\overline{S}_r$, such that $\pi(S_s,S_r) \leqslant 0$ for all $S_s \geqslant \overline{S}_s$ and $S_r \geqslant \overline{S}_r$.*

Proof. See Appendix B. □

Due to the complexity of the problem, it is intractable to derive the analytical solutions for the optimal base-stock levels. However, Lemma 1 ascertains that with sufficiently large values as heuristic upper bounds, the optimal base-stock levels can be identified by applying complete enumeration within the bounds.

3.2. The competitive (decentralized) channel: Nash equilibrium

When the supply chain is decentralized, the supplier and the retailer are independent decision makers, and each looks at its own welfare when making stocking decisions, ignoring the collective impact on the supply chain as a whole. Under the decentralized system, it is straightforward to verify that the steady-state expected profits for the supplier and the retailer, respectively, can be represented as functions of S_s and S_r as

$$\pi_s(S_s,S_r) = m_d Q_d^{(S_s,S_r)} + \delta m_r Q_r^{(S_s,S_r)} - h_s I_s^{(S_s,S_r)}, \tag{15}$$

$$\pi_r(S_s,S_r) = (1-\delta) m_r Q_r^{(S_s,S_r)} - h_r I_r^{(S_s,S_r)}, \tag{16}$$

where δ, a measure of the degree of double marginalization, is the fraction of the retail margin that is allocated to the supplier:

$$\delta = \frac{w - c_r}{r - c_r}; \quad 0 \leqslant \delta \leqslant 1. \tag{17}$$

Note that the cost of shipping the product from the supplier to the retailer is assumed to be borne by the supplier and is thus included in c_r. However, the suppler can easily transfer this transportation cost to the retailer by increasing the wholesale price w to accommodate this cost. In this case, the degree of double marginalization δ would be relatively higher according to (17). Assume that, with the objective of optimizing their own profits, the supplier and the retailer simultaneously choose their respective base-stock levels, S_s and S_r, in the game's only move, and the stocking decisions are continuously committed by the two firms over an infinite horizon. Let $\boldsymbol{\sigma}_s$ be the supplier's strategy profile and $\boldsymbol{\sigma}_r$ be the retailer's strategy profile, and let $\boldsymbol{\sigma}_s = \boldsymbol{\sigma}_r = \{S \in \mathbf{Z} | 0 \leqslant S \leqslant \bar{S}\}$, where $\bar{S}$ is a very large constant. Then, the best reaction mappings for the supplier and the retailer, respectively, are given by

$$\Phi_s(S_r) = \left\{ S_s \in \boldsymbol{\sigma}_s | \pi_s(S_s, S_r) = \max_{S \in \boldsymbol{\sigma}_s} \pi_s(S, S_r) \right\}, \tag{18}$$

$$\Phi_r(S_s) = \left\{ S_r \in \boldsymbol{\sigma}_r | \pi_r(S_s, S_r) = \max_{S \in \boldsymbol{\sigma}_r} \pi_r(S_s, S) \right\}. \tag{19}$$

A pure strategy Nash equilibrium is a pair of base-stock levels, $(\widehat{S}_s, \widehat{S}_r)$, such that each firm's stocking decision is a best response to the other's:

$$\widehat{S}_s \in \Phi_s(\widehat{S}_r) \quad \text{and} \quad \widehat{S}_r \in \Phi_r(\widehat{S}_s). \tag{20}$$

Similar to the centralized problem, we can apply complete enumeration within the strategy profiles to numerically find the Nash equilibria with the conditions specified in (18)–(20). For illustrative purposes, in Table 1 the two firms' best reaction functions are plotted using a set of parametric values, and the resulting Nash solution is compared to the system optimal solution. Note that the discrete nature of the problem and its intricacy make it difficult to analytically ascertain the existence and the uniqueness of Nash equilibrium. However, after examining a variety of parametric values, we find that a pure strategy Nash equilibrium exists uniquely in most scenarios. Exceptional cases with no equilibrium or multiple equilibria are very rare. As exemplified in Fig. 4, such cases occur due to discrete effect of base-stock levels. They do not appear to affect our qualitative investigation of the problem.

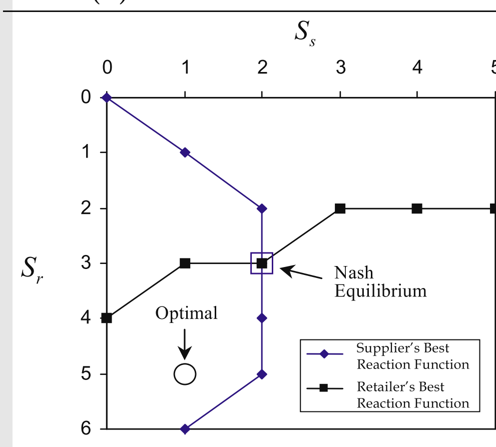
Following most studies which model inventory competition in stochastic settings (e.g., Cachon and Zipkin, 1999; Axsäter, 2001), our model assumes that the supplier and the retailer react to base-stock levels instead of the actual stock levels. In fact, it is ambiguous how each player can react to the other player's actual stock levels. The underlying assumption to sustain the Nash equilibrium of actual

Table 1
System optimal solution vs. Nash solution.

(a) Base Parametric Values

Demand Rates:	$\lambda = 15$
	$\alpha = 0$
Replenishment Rates:	$\mu_s = 5$
	$\mu_r = 10$
Unit Holding Costs:	$h_s = 150$
	$h_r = 200$
Unit Margins:	$m_r = m_d = 100$
Double Marginalization:	$\delta = 0.5$
Substitution Rates:	$\beta_r = \beta_d = 0$

(b) Best Reaction Functions

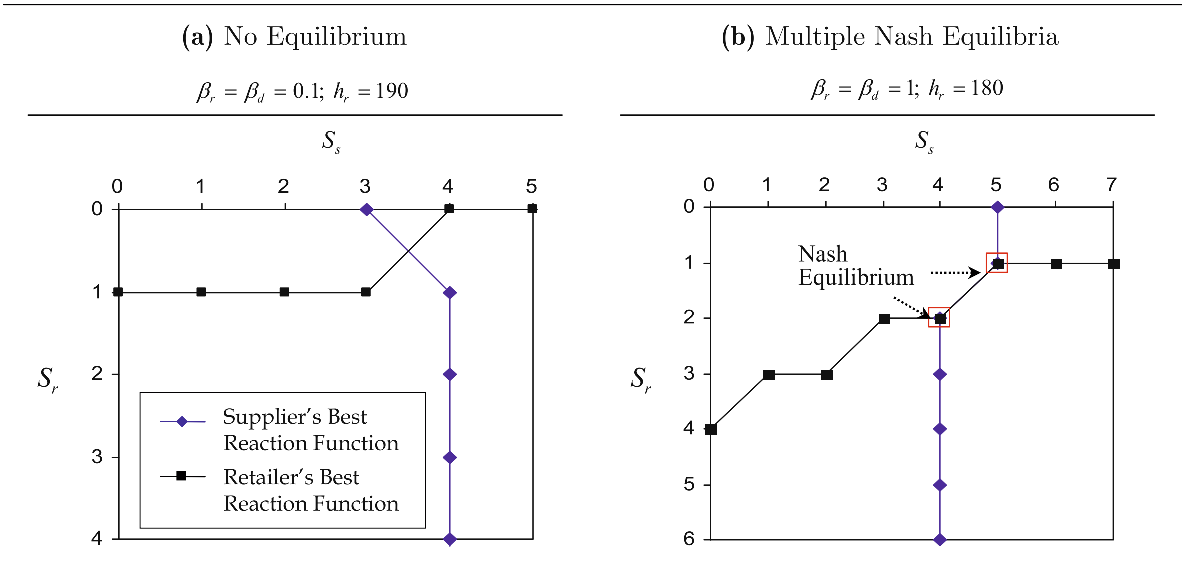


(c) Cooperative Profit Table (π)

S_r \ S_s	0	1	2	3	4	5
0	0	-150	-300	-450	-600	-750
1	236	354	341	238	98	-50
2	445	589	649	622	527	396
3	617	735	787	771	696	580
4	744	815	826	781	692	573
5	817	**831**	789	704	589	456
6	830	785	694	572	434	289

(d) Competitive Profit Table (π_s, π_r)

S_r \ S_s	0	1	2	3	4	5
0	0, 0	-150, 0	-300, 0	-450, 0	-600, 0	-750, 0
1	136, 100	185, 169	136, 205	21, 217	-121, 220	-270, 220
2	264, 181	347, 242	365, 284	313, 309	208, 319	73, 323
3	380, 237	464, 271	**497, 290**	471, 301	391, 306	273, 308
4	482, 261	555, 259	583, 243	556, 225	481, 211	368, 204
5	568, 249	625, 206	639, 151	603, 101	522, 67	408, 48
6	634, 196	674, 111	674, 20	628, -56	540, -106	422, -134



Note. Unless otherwise noted, the base parametric values in Table 1(a) are used.

Fig. 4. Examples of no equilibrium and multiple equilibria.

stock levels is that the payoffs for each one-shot event are predictable. However, ascertaining the payoffs in such a setting (regardless of whether they decide to replenish or not) is challenging for at least two reasons. First, the players may not know each other's actual stock levels. Second, more importantly, even if the information of actual stock levels is shared, given that the demand process and the replenishment lead time are stochastic in nature, the exact arrival times for the next customer and the replenishment order are unknown. The studies cited above reflect a uniform view that the Nash equilibria of base-stock levels can be sustained if the game is played over the infinite horizon such that the expected payoffs become common knowledge. Strictly speaking, recognizing that repeated interactions can result in other equilibria than the Nash equilibria of the single-period (constituent) game, we seem to need to impose an assumption that the players will commit to a base-stock decision throughout their interactions. Yet, in some sense, this assumption is not required since the incentives for the players to deviate from their base-stock decisions are unclear for the same reasons mentioned earlier. Besides, if the game is finite but is long enough for determining the expected payoffs, the Folk theorem implies that there is a unique equilibrium which simply is the identical repetition of the unique Nash equilibrium of the constituent game.

4. Numerical analysis of competition effect

To investigate the impact of vertical and horizontal competition on the firms' stocking behaviors and the supply chain efficiencies, we developed a FORTRAN program to perform a numerical study based on the models specified above. In the program, subroutines LSLRG and LFSRG in the IMSL MATH/LIBRARY are utilized to solve the system of linear algebraic equations in (9). Specifically, LSLRG first computes an **LU** factorization of the coefficient matrix $\mathbf{A}^{(S_s,S_r)}$ based on Gauss elimination with partial pivoting. The steady-state probabilities $\mathbf{P}^{(S_s,S_r)}$ is then found by LFSRG which solves the lower triangular system $\mathbf{Ly} = \mathbf{0}$ using forward elimination and the upper triangular system $\mathbf{UP}^{(S_s,S_r)} = y$ using backward substitution. While there are numerous scenarios for the numerical experiments, we found that certain parametric values generate similar qualitative results in terms of system behavior. Therefore, we avoid redundancy by choosing the set of parametric values given in Table 1a as the base parametric values for our numerical study.

4.1. Effect of double marginalization

Supply chain inefficiency as a result of double marginalization occurs when the manufacturer and retailer act independently and each only receives a portion of the total contribution margin. As mentioned above, it is recognized in the supply chain literature that vertical competition induced by double marginalization may cause understocking behavior within a distribution channel. Now we touch on the similar issue under the dual-channel context.

Table 2 reports the computational results of the analysis based on the base parametric values. Different values of α (the proportion of direct customers) ranging from 0 to 1 with step value 0.25, and different values of δ (the degree of double marginalization) ranging from 0 to 1 with step value 0.1 are used in the analysis. Note that $\delta = 0$ implies no double marginalization ($w = c$), while $\delta = 1$ indicates the highest degree of double marginalization ($w = p_r$). In order to measure the channel efficiency, we define the *competition penalty*, as the difference in supply chain profit between a Nash solution and the system optimal solution, measured as a percentage of the optimal profit. In Table 2, the competitive stock levels in equilibrium, the optimal stock levels and the corresponding competition penalty are reported for each α and δ. We first make the following observation:

Result 1. With a low proportion of direct customers, increasing the degree of double marginalization intensifies understocking behavior of the whole channel, and thus increases the competition penalty.

Table 2
Impact of double marginalization on channel efficiency.

Degree of Double Marginalization	α = 0.00			α = 0.25			α = 0.50			α = 0.75			α = 1.00		
	Equilibrium		Competition Penalty	Equilibrium		Competition Penalty	Equilibrium		Competition Penalty	Equilibrium		Competition Penalty	Equilibrium		Competition Penalty
δ	$\hat{S}_s$	$\hat{S}_r$		$\hat{S}_s$	$\hat{S}_r$		$\hat{S}_s$	$\hat{S}_r$		$\hat{S}_s$	$\hat{S}_r$		$\hat{S}_s$	$\hat{S}_r$	
0.0	1	5	0.0%	3	3	0.0%	4	2	0.0%	4	1	0.0%	5	0	0.0%
0.2	1	4	1.9%	3	2	1.6%	4	2	0.0%	4	1	0.0%	5	0	0.0%
0.4	2	3	5.2%	3	2	1.6%	4	1	6.8%	4	1	0.0%	5	0	0.0%
0.6	2	2	21.9%	3	2	1.6%	4	1	6.8%	4	0	11.9%	5	0	0.0%
0.8	1	1	57.4%	3	1	28.2%	3	0	44.6%	4	0	11.9%	5	0	0.0%
1.0	0	0	100.0%	1	0	81.6%	3	0	44.6%	4	0	11.9%	5	0	0.0%
Optimal:	**1**	**5**		**3**	**3**		**4**	**2**		**4**	**1**		**5**	**0**	

Note. (1) α = Proportion of Direct Customers
(2) Unless otherwise noted, the base parametric values in Table 1(a) are used.
(3) Shaded area indicates that the competitive base-stock levels perform at the optimal level in equilibrium.

We remark that when $\alpha = 0$ (no direct customer in the system), the problem reduces to the traditional two-echelon supply chain and our result is consistent with the findings in the vertical competition literature. Although the supplier may overstock when the degree of double marginalization δ is moderate, the total inventory level of the supply chain as a whole understocks in most cases. On the other hand, when $\alpha = 1$, all customers are direct customers and the channel in effect is "integrated." Not surprisingly, there is no competition penalty for all δ in this extreme case. Intuitively, this explains the result below:

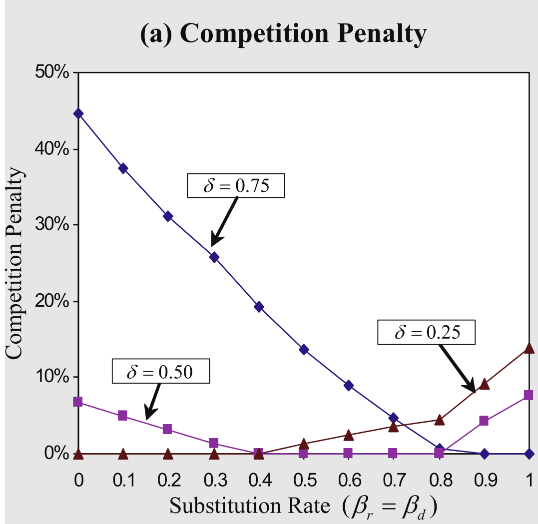
Result 2. For any given degree of double marginalization, the system with a higher proportion of direct customers results in a lower competition penalty.

In this section, without considering the effect of horizontal competition, we focus our analysis on the effect of vertical competition induced by double marginalization. What happens if customers' stock-out based substitution occurs in the dual-channel distribution? Next, we examine the combined effect of vertical competition and horizontal competition.

4.2. Combined effect of vertical and horizontal competition

The model specifies that when a stock-out occurs in either channel, customers will shift to the other channel with a known probability. Recall that we defined β_r (β_d) as the proportion of the retail (direct) customers who will switch to the direct (retail) channel when a stock-out occurs. To investigate the effect of stock-out based substitution on channel efficiency when it co-exists with double marginalization, we first assume that the substitution rates are symmetry across the two channels, i.e., $\beta_r = \beta_d$.

Table 3
Impact of stock-out based substitution.



(b) Base Stock Levels

Substitution Rate	Proportion of Direct Customers $\alpha = 0.5$							
	Optimal		Equilibrium					
	δ = 0		δ = 0.25		δ = 0.50		δ = 0.75	
$(\beta_r = \beta_d)$	S_s^*	S_r^*	$\hat{S}_s$	$\hat{S}_r$	$\hat{S}_s$	$\hat{S}_r$	$\hat{S}_s$	$\hat{S}_r$
0.0	**4**	**2**	4	2	4	1	3	0
0.2	**4**	**2**	4	2	4	1	3	0
0.4	**4**	**1**	4	2	4	1	4	0
0.6	**4**	**1**	4	2	4	1	4	0
0.8	**4**	**1**	4	2	4	1	5	0
1.0	**5**	**0**	4	2	5	1	5	0

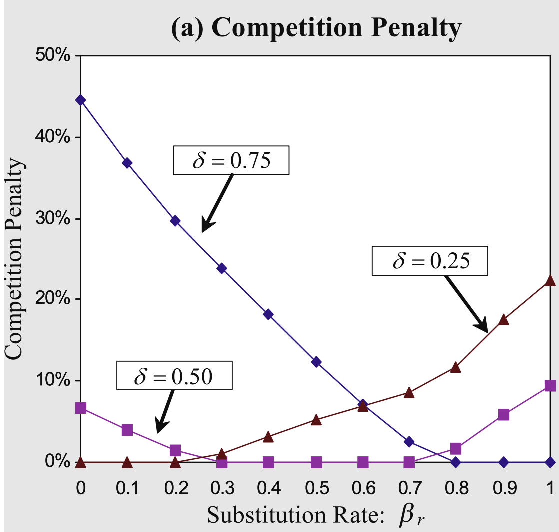
Note. (1) δ = Degree of Double Marginalization.
(2) Unless otherwise noted, the base parametric values in Table 1(a) are used.
(3) Shaded area indicates that the competitive base-stock levels perform at the optimal levels in equilibrium.

We consider three scenarios based on different degrees of double marginalization (low: $\delta = 0.25$; moderate: $\delta = 0.50$; high: $\delta = 0.75$) and examine the effect of increasing stock-out based substitution rates for each scenario. The outcome is summarized in Table 3, and we observe that:

Result 3. Simultaneously increasing stock-out based substitution rates of retail and direct customers diminishes/intensifies the competition penalty when the degree of double marginalization is high/low.

Result 3 provides some insight into the combined effect of vertical and horizontal competition on the channel efficiency under the assumption that $\beta_r = \beta_d$. The independent effect of β_r and β_d, respectively, is yet ambiguous. In reality, given that the two marketing channels are different with respect to their characteristics, it is hard to imagine that the values of β_r and β_d are identical. In other words, when a stock-out occurs, the proportion of retail customers who will search and switch to the other channel is often different from that of direct customers. Consequently, the asymmetric picture deserves further investigation. In recognition of this motivation, Tables 4 and 5, respectively, summarize the individual impact of β_r and β_d on the stocking behaviors and the channel efficiency. By comparing the two tables, the immediate result follows:

Table 4
Impact of substitution rate β_r.

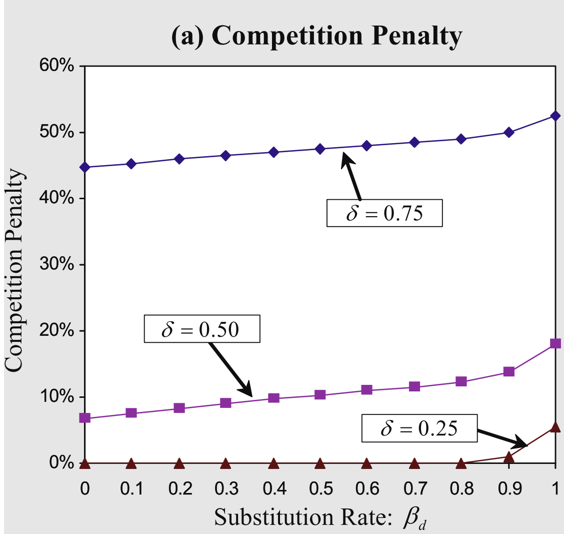


(b) Base Stock Levels

Substitution Rate	Proportion of Direct Customers $\alpha = 0.5$							
	Optimal		Equilibrium					
	δ = 0		δ = 0.25		δ = 0.50		δ = 0.75	
β_r	S_s^*	S_r^*	$\hat{S}_s$	$\hat{S}_r$	$\hat{S}_s$	$\hat{S}_r$	$\hat{S}_s$	$\hat{S}_r$
0.0	**4**	**2**	4	2	4	1	3	0
0.2	**4**	**2**	4	2	4	1	3	0
0.4	**4**	**1**	4	2	4	1	4	0
0.6	**4**	**1**	4	2	4	1	4	0
0.8	**5**	**0**	4	2	5	1	5	0
1.0	**5**	**0**	4	2	5	1	5	0

Note. (1) δ = Degree of Double Marginalization.
(2) Unless otherwise noted, the base parametric values in Table 1(a) are used.
(3) Shaded area indicates that the competitive stock levels perform at the optimal levels in equilibrium.

Table 5
Impact of substitution rate β_d.



(b) Base Stock Levels

Substitution Rate	Proportion of Direct Customers $\alpha = 0.5$							
	Optimal		Equilibrium					
	δ = 0		δ = 0.25		δ = 0.50		δ = 0.75	
β_d	S_s^*	S_r^*	$\hat{S}_s$	$\hat{S}_r$	$\hat{S}_s$	$\hat{S}_r$	$\hat{S}_s$	$\hat{S}_r$
0.0	**4**	**2**	4	2	4	1	3	0
0.2	**4**	**2**	4	2	4	1	3	0
0.4	**4**	**2**	4	2	4	1	3	0
0.6	**4**	**2**	4	2	4	1	3	0
0.8	**3**	**3**	4	2	4	1	3	0
1.0	**0**	**6**	4	2	4	1	3	0

Note. (1) δ = Degree of Double Marginalization.
(2) Unless otherwise noted, the base parametric values in Table 1(a) are used.
(3) Shaded area indicates that the competitive base-stock levels perform at the optimal levels in equilibrium.

Result 4. The effect of stock-out based substitution rates of retail customers on the channel efficiency is more substantial than that of direct customers.

Some justification of this result will be provided later on. To gain more insights, we now look into Tables 4 and 5 in more details.

When the degree of double marginalization is high ($\delta = 0.75$), the decentralized supply chain suffers from the inefficiency caused by competitive understocking. We find that, depending on the type of asymmetry of stock-out based substitution, the effect of combined vertical–horizontal competition may increase or decrease the channel efficiency. Specifically, if stock-out based substitution rate of retail customers dominates that of direct customers, horizontal competition may help to improve supply chain performance by counterbalancing the understocking behaviors at both the upper and lower echelons (see Table 4). If it is the other way around, however, the effect of combined vertical–horizontal competition is detrimental to the channel efficiency since the understocking behaviors caused by the effect of vertical competition may be aggravated by the effect of horizontal competition (see Table 5). In summary, we find that:

Result 5. When the degree of double marginalization is high, increasing stock-out based substitution rate of retail customers improves the channel efficiency, while conversely, increasing that of direct customers deteriorates the channel efficiency.

On the other hand, when the degree of double marginalization is low ($\delta = 0.25$), the competitive dual-channel supply chain is relatively more efficient. In this case, increasing stock-out based substitution of either retail or direct customers or both typically leads to a higher competition penalty. This result is outlined below:

Result 6. Regardless of the type of asymmetry of stock-out based substitution, the effect of combined vertical–horizontal competition tends to decrease the channel efficiency when the degree of double marginalization is low.

Observant readers may have noticed that the competitive stocking levels in equilibrium are insensitive to customers' stock-out based substitution. Before getting into the details to understand the reason behind, we formalize this observation below:

Result 7. While the integrated supplier–retailer may reduce or/and consolidate the based-stock levels to benefit from stock-out based substitution through the risk pooling effect, the independent supplier and retailer are more inattentive to customers' stock-out based substitution.

We first focus on the change in the value of β_r to explain this result. From Table 4b, we learn that if there were sufficient amount of retail customers willing to switch to the direct channel, the integrated supplier–retailer would lower S_r to increase the probability of stock-out at the retailer and decrease that at the supplier (see Fig. 5a:R1). This would allow the system to benefit from stock-out based substitution through the risk pooling effect. Nonetheless, in the competitive supply chain, reducing S_r would make the independent retailer worse off since it increases the stock-out probability at the retailer (more lost sales). As a result, the self-interested retailer would be vigorously averse to do so. Under this circumstance, it would not be beneficial for the supplier to increase S_s unless the additional inventory could appropriate enough retail customers. As evident in Table 4b, this happens only when β_r and δ are high enough (a high β_r diverts more retail customers to the direct channel in case of a stock-out, while a high δ causes the retailer to understock and thus increases the stock-out probability at the retailer). Even though the supplier may raise S_s, the action would hardly cause the retailer to adjust S_r since its impact on the stock-out probability at the retailer is not considerable (see Fig. 5b:R2). Thus, the competitive stocking levels in equilibrium are insensitive to β_r.

In response to an increase in β_d, Table 5b indicates that the integrated supplier–retailer may also consolidate the based-stock levels to capture the risk pooling benefit. It should be noticed that, when its value is not high, the impact of β_d is not as significant as that of β_r. The reason being is that, unlike reducing S_r, reducing S_s would increase the probability of stock-out not only at the supplier, but also at the retailer (compare Fig. 5b:R1 with a:R1). This means that the retailer may not have enough inventories to absorb more customers diverted from the direct channel due to a stock-out. In this case, in order to reduce the probability of stock-out at the retailer, S_r needs to be increased. However, since the inventory holding cost at the retailer is relatively higher, it may not be lucrative to do so until β_d is sufficiently high (as evident in Table 5b). We note that the justification here also elucidates Result 4 to some extent. On the other hand, when

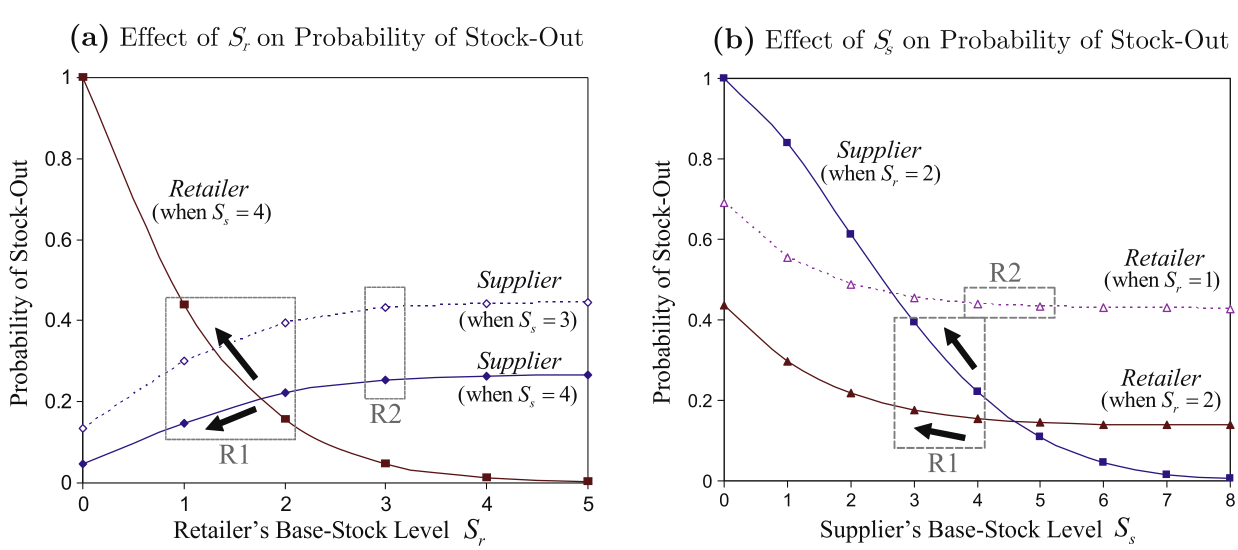


Fig. 5. Probability of stock-out. (a) Effect of S_r on probability of stock-out and (b) effect of S_s on probability of stock-out.

the supply chain is decentralized, in order to secure the demand from direct customers, the self-interested supplier would be reluctant to lower S_s even when β_d is extremely high. Without S_s being reduced, increasing S_r would hardly make the independent retailer better off since the stock-out probability at the supplier is too low (see Fig. 5a:R2) to divert sufficient direct customers to the retailer. This explains why the impact of β_d on the competitive stocking levels is not significant.

5. Channel coordination

The result in the previous section indicates that, in equilibrium, the competitive base-stock levels of the two-echelon dual-channel supply chain rarely agree with the system optimal base-stock levels. In other words, the decentralized supply chain is inefficient in most cases.

Can the incentives of the two channel entities be aligned through a contract such that the supply chain performs at the optimal level in equilibrium? In this section, we address this question by investigating how contractual arrangements can coordinate the stocking decisions of the two channel entities in a system optimal fashion.

Obviously, if each firm's profit function can be reformulated through a contract such that it becomes a linear transformation of the centralized profit function, then the contract will be able coordinate the channel in any scenario as the firms have the same objective of maximizing the total channel profit. In light of this discernment, we develop a contract called "inventory and direct revenue sharing," or "$k(\rho,\theta)$-contract" for brevity, to see how to put it in practice.

Under the $k(\rho,\theta)$-contract, the two firms agree to share the total supply chain inventory holding cost by making transfer payments such that the supplier incurs some fraction ρ of the cost, and the retailer incurs the remaining fraction $1-\rho$ of the cost. Moreover, the supplier agrees to transfer some fraction θ of the direct sales revenue to the retailer. Specifically, to attain this agreement in practice, the supplier transfers k_s to the retailer per unit time, while the retailer transfers k_r to the supplier per unit time, where

$$k_s = \theta m_d Q_d^{(S_s,S_r)} + (1-\rho) h_s I_s^{(S_s,S_r)} \quad \text{and} \quad k_r = \rho h_r I_r^{(S_s,S_r)}. \tag{21}$$

In effect, the profit functions for the supplier and the retailer with a $k(\theta,\rho)$-contract, respectively, are

$$\pi_s^k(S_s,S_r) = (1-\theta) m_d Q_d^{(S_s,S_r)} + \delta m_r Q_r^{(S_s,S_r)} - \rho[h_s I_s^{(S_s,S_r)} + h_r I_r^{(S_s,S_r)}], \tag{22}$$

$$\pi_r^k(S_s,S_r) = \theta m_d Q_d^{(S_s,S_r)} + (1-\delta) m_r Q_r^{(S_s,S_r)} - (1-\rho)[h_s I_s^{(S_s,S_r)} + h_r I_r^{(S_s,S_r)}]. \tag{23}$$

The choice of θ and ρ can be arbitrary. However, if we set $\theta = 1-\delta$ and $\rho = \delta$, then the $k(\theta,\rho)$-contract modifies the profit functions in (22) and (23), respectively, to

$$\pi_s^k(S_s,S_r) = \delta\pi(S_s,S_r), \tag{24}$$

$$\pi_r^k(S_s,S_r) = (1-\delta)\pi(S_s,S_r), \tag{25}$$

where $\pi(S_s,S_r)$ is the centralized profit function defined in (14). As the two firms face a profit function that is proportional to the centralized one, such an arrangement in effect impersonates a profit sharing mechanism (Jeuland and Shugan, 1983), and thus, there must exist a Nash equilibrium that is congruent with the system optimal base-stock levels.

Note that the $k(\theta,\rho)$-contract may not always result in Pareto improvement, as it does not provide a degree of freedom in splitting the total channel profit. As a result, periodic fixed transfer payments may be required to reallocate the gains from coordination. One might argue that a more straightforward way to achieve channel coordination is to provide a contract under which transfer payments are done so that retailer gets a fraction and the supplier gets the remaining fraction of the total channel profit. However, a wary firm may circumvent this sort of agreement since it requires the channel members' profits to be credibly revealed. In fact, it is hardly imaginable that firms would transparently disclose their actual profit under a competitive environment, especially when they are not convinced that their allocated share of profit is fair. Thus, a contract, such as the $k(\theta,\rho)$-contract, that provides a guidance as to what data should be monitored and verified may still have to be resorted to.

The implementation of $k(\rho,\theta)$-contract is not without a cost. It entails gathering the inventory data either through an information system that continuously tracks the inventory levels of the two stocking locations, or by a manual process of visiting each stocking location and conducting a physical count, possibly via scanners. In addition, the retailer may need to screen the supplier's online point of sale (POS) through, for example, an Internet technology. Needless to say, the $k(\rho,\theta)$-contract would not be practical if the economic advantage of implementing it fails to offset the cost associated with establishing procedures to verify compliance.

While an appropriately designed $k(\rho,\theta)$-contract warrants the optimal channel performance in any circumstance, the implementation hassle may cause it to be undesirable. Hence, we must ask a question: is there any robust coordination contract that is simple and more intuitive to implement? Unfortunately, our investigation indicates that the co-existence of both vertical and horizontal competition in the dual-channel distribution makes it difficult to identify such a contract. Nevertheless, there are several contracts in the supply chain literature that are relatively unsophisticated and may be used as alternatives to mitigate the issue. Although these contracts do not coordinate the dual-channel supply chain for all possible scenarios, they can be effective if firms know "when to use what." We identify a few of such contracts and exemplify them in Table 6.

Specifically, with an inventory cost subsidy contract, the supplier pays the retailer a per unit holding cost of retail inventory at a constant rate. An inventory cost subsidy contract, which induces the retailer to carry more inventories, is analogous to a buy-back contract (Pasternack, 1985) under which the supplier agrees to buy-back some portion of the retailer's unsold inventory. Pasternack (1985) shows that, in a hierarchical supply chain with no direct distribution, channel coordination can be achieved by a buy-back contract. However, in the dual-channel supply chain, Table 6 indicates that an inventory cost subsidy contract is able to coordinate the channel only under some scenarios (3, 5 and 7) wherein the retailer understocks. In scenario 6, an inventory cost subsidy contract motivates the retailer to stock at the efficient level, but it fails to restrain the supplier's overstocking behavior. In such case, the contract has to be combined with a retail revenue sharing contract to attain channel coordination. With a retail revenue sharing contract, the retailer makes payments to the supplier based upon the retail sales revenue. Dana and Spier (2001) show that a retail revenue sharing contract is a valuable instrument for the upstream firm to soften downstream horizontal competition and encourage efficient inventory holding in a decentralized supply chain. In

Table 6
Selecting coordination contracts.

Competitive Scenario	1	2	3	4	5	6	7	8
Main Effect Customer Type	Retail Customers				Direct Customers			
Stock-Out Based Substitution	High		Low		High		Low	
Double Marginalization	High	Low	High	Low	High	Low	High	Low
Stocking Behavior								
Supplier	E	U	U	E	U	O	O	E
Retailer	E	O	U	E	U	U	U	E
Channel Contract								
Inventory Cost Subsidy (ICS)			Y		Y		Y	
Retail Revenue Sharing (RRS)		Y						
ICS and RRS						Y		

Note. E = Efficient; U = Understock; O = Overstock; Y = The contract coordinates the channel in a system optimal fashion.

the dual-channel supply chain, our result shows that, in some certain cases (scenarios 2 and 6), this contract may help to coordinate the supply chain by discouraging the retailer or the supplier from carrying too many inventories. Note that when the supply chain is coordinated by either a holding cost subsidy contract or a retail revenue sharing contract, the contract may not always result in Pareto improvement. Therefore, in order to be able to implement the contract, periodic fixed transfer payments, such as franchise fees, may be required to reallocate the gains from coordination.

6. Model extension: Stackelberg Leader–Follower games

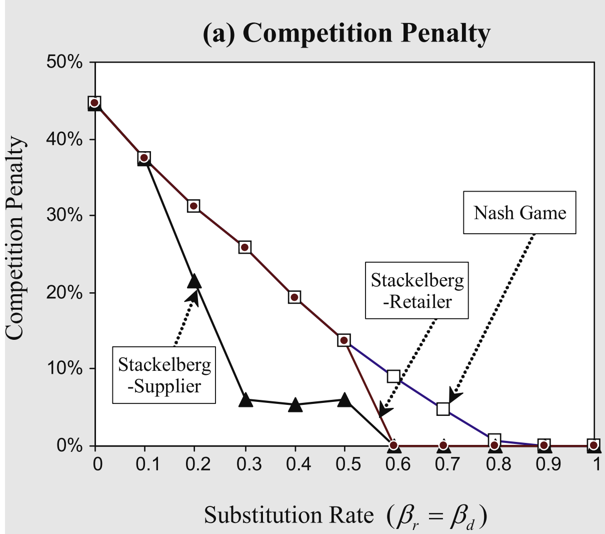
In this paper, we assume that both the supplier and the retailer are blind to how their own decision on the base stock level affects their partner's decision. In other words, there is no leadership in the distribution channel and the setting yields a Nash equilibrium. What happens when channel leadership emerges? Could channel leadership benefit the dual-channel distribution in terms of improving channel efficiency? To answer the questions, we further examine two different Leader–Follower games: Stackelberg-Supplier game and Stackelberg-Retailer game.

In the Stackelberg-Supplier game, the supplier knows the retailer's best reaction function when determining its base-stock level. The supplier announces the inventory decision first and the retailer, as a follower, reacts rationally to the announcement by choosing its base-stock level. The Stackelberg-Supplier equilibrium corresponds to the solution of the optimization problem:

$$\underset{(S_s,\widehat{S}_r)\in\sigma_s\times\sigma_r}{\text{maximize}}\ \pi_s(S_s,\widehat{S}_r), \tag{26}$$

$$\text{subject to}\ \widehat{S}_r \in \underset{S_r\in\sigma_r}{\operatorname{argmax}}\ \pi_r(S_s,S_r), \tag{27}$$

Table 7
Competition penalty for the Stackelberg games and the vertical Nash game.



(b) Base Stock Levels

Substitution Rate	Optimal		Equilibrium					
					Stackelberg Leader			
			Nash		Supplier		Retailer	
$(\beta_r = \beta_d)$	S_s^*	S_r^*	$\widehat{S}_s$	$\widehat{S}_r$	$\widehat{S}_s$	$\widehat{S}_r$	$\widehat{S}_s$	$\widehat{S}_r$
0.0	**4**	**2**	3	0	3	0	3	0
0.2	**4**	**2**	3	0	2	1	3	0
0.4	**4**	**1**	4	0	3	1	4	0
0.6	**4**	**1**	4	0	4	1	4	1
0.8	**4**	**1**	5	0	4	1	4	1
1.0	**5**	**0**	5	0	5	0	5	0

Note. (1) $\alpha = 0.5$; $\delta = 0.75$.
(2) Unless otherwise noted, the base parametric values in Table 1(a) are used.
(3) Shaded area indicates that the competitive base-stock levels perform at the optimal levels in equilibrium.

where argmax is defined as an argument that maximizes the function that follows and the profit functions, $\pi_s(S_s,S_r)$ and $\pi_r(S_s,S_r)$, are given in (15) and (16), respectively.

In a converse fashion, the meaning of the Stackelberg-Retailer game is obvious. The subsequent equilibrium of the game can be obtained by solving the problem below:

$$\underset{(\widehat{S}_s,S_r)\in\sigma_s\times\sigma_r}{\text{maximize}}\ \pi_r(\widehat{S}_s,S_r), \tag{28}$$

$$\text{subject to } \widehat{S}_s \in \underset{S_s\in\sigma_s}{\operatorname{argmax}}\ \pi_s(S_s,S_r). \tag{29}$$

The outcomes from different game scenarios are juxtaposed in Table 7 and it is interesting to observe the following result:

Result 8. Stackelberg channel leadership by either the supplier or the retailer can improve the channel efficiency in the dual-channel distribution.

This result substantiates statements within channel literature which advocates the introduction of a ‘‘chain captain” as coordinator in a channel (Jørgensen et al., 2001).

7. Concluding remarks

The trend of engaging in Internet based direct sales has raised serious awareness and attention, by academicians and practitioners, to the opportunities and challenges of using both direct and non-direct distribution channels concurrently. Given the significance of this subject, the objective of this paper is to conceptualize the strategic interactions among channel entities with respect to their decisions of product availability in a dual-channel distribution. We reveal the nature of channel conflict caused by simultaneous vertical and horizontal competition in a dual-channel supply chain by investigating the impact of customers' stock-out based substitution on the channel efficiency.

This paper makes a number of contributions to the literatures on channels of distribution. First, in terms of modeling and analysis, we model the first stock availability game of a distribution channel in a dual-channel context. With the trend of adopting a dual-channel distribution strategy in the recent business environment, the channel distribution system wherein the upstream supply chain member is both a supplier and a competitor of the downstream supply chain member is not uncommon. Since the theoretical basis for the analysis of competitive product availability in a dual-channel distribution has not yet been developed, our game-theoretical model, wherein stock competition is among firms located both within the same echelon and in different echelons, can be viewed as the primitive prototype in approaching this subject. Second, with respect to insights and implications on channels of distribution, we provide circumstances where the effect of combined vertical–horizontal competition may increase or decrease the channel efficiency in a dual-channel supply chain. Conventional wisdom suggests that the effect of combined vertical–horizontal competition may help to improve supply chain efficiency. We show that this result is not always applicable in a dual-channel supply chain where asymmetry enters the picture. Third, regarding guidelines for channel coordination, we investigate various channel contracts to shed light on coordination issues in a dual-channel supply chain. Finally, we show that the emergence of channel leadership can benefit the dual-channel distribution in terms of improving channel efficiency.

The focus of this study is on issues related to competitive inventory decisions. Thus, we implicitly assume that the supplier is committed to the dual-channel distribution. However, our model can be easily extended to address the channel design problem by comparing the performance of three channel strategies: dual-channel, retail-only, and direct-only strategies. Specifically, if the direct channel is dropped, then β_d becomes the proportion of direct customers who will buy at the retail store due to the absence of the direct channel, and β_r becomes zero since no substitute channel is available. As a result, the arrival rates of direct and retail customers become $\lambda_d = 0$ and $\lambda_r = [\beta_d a + (1-\alpha)]\lambda$, respectively. With the modification, the performance of the retail-only strategy can be evaluated. On the other hand, to evaluate the performance of the direct-only strategy, we can simply set the base-stock level S_r to zero and follow the same analysis.

We recognize that our model, which appears to foreshadow future research extensions, is limited in many respects. For example, due to the complex nature of the problem, we consider only one supplier and one retailer in the dual-channel distribution. Albeit such a setting provides an appropriate starting point for investigating the problem, it should be valuable to extend the analysis by exploring different channel structures. Another potential restriction of this paper is that the model assumes a one-for-one replenishment policy. While this stylized inventory model is justifiable for our insight-oriented investigation, studies seeking to tackle the problem with different inventory policies may be warranted.

Consistent with the extant studies investigating the echelon inventory games in multi-period settings (e.g., Cachon and Zipkin, 1999; Cachon, 2001), we focus the analysis on inventory competition and isolate the effect from price competition by treating the prices as exogenous variables. Apparently, rational ambiguity and analytical tractability are the reasons behind this treatment. For a typical two-level decentralized supply chain, price decisions are made by the manufacturer and the retailer in a sequential manner, while in contrast, the inventory decisions are made simultaneously between the two channel members in the multi-period environment. If the channel members concurrently make the price and inventory decisions, it is dubious whether the sub-game perfect equilibrium can be obtained because backward induction cannot be applied to games of imperfect information (presumably, the channel members would not truthfully reveal their inventory strategies in the first stage of the game). Another reason for the absence of price decisions in the extant literature of echelon inventory games is that companies typically make inventory decisions after wholesale and retail prices are set. Although prices can still be changed, short-term price adjustments are not frequently observed in most cases. For this reason, it appears to be more plausible to separate the analysis of inventory decisions from that of price decisions. We remark that in the multi-channel context, the absence of the price competition between the direct and the retail channels is empirically evident. As pointed out by Tsay and Agrawal (2004a), one common strategy for the manufacturer to minimize the conflict and enable co-existence of competing sales channels is to avoid price competition between the direct and the retail channels. According to a survey by Ernst and Young (2001), nearly two thirds of firms price products identically in their online and offline stores. Indubitably, isolating the effect from price competition enables us to provide realistic results that add to the limited literature on the subject. Having said that, we recognize that the validation of model assumption has to trump the tractability consideration when price competition co-exists with inventory competition. Obviously, a novel approach is compelled to shed light on the case.

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Appendix A. Proof of Lemma 1

(i) Let $D(t)$ and $R(t)$, respectively, denote the number of direct and retail customers in time interval $[0,t]$. Since the aggregate demand follows a Poisson process $\{N(t), t \geqslant 0\}$, it follows that $N(t) = D(t) + R(t) = k \in \mathbb{N}$, and

$$p\{N(t) = k\} = \frac{e^{-\lambda t}(\lambda t)^k}{k!}. \tag{A1}$$

Given that there are k arrivals in $[0,t]$, the probability of observing m direct customers and n retail customers (note that $k = m + n$) is a binomial probability given by

$$p\{D(t) = m, \quad R(t) = n | N(t) = k\} = \binom{k}{m} \alpha^m (1-\alpha)^n. \tag{A2}$$

Based on equations (A1) and (A2), the probability of observing m direct customers in time interval $[0,t]$ can be specified as

$$\begin{aligned}
p\{D(t) = m\} &= \sum_{n=0}^{\infty} p\{D(t) = m, R(t) = n\} \\
&= \sum_{n=0}^{\infty} p\{D(t) = m, R(t) = n | N(t) = k\} \times p\{N(t) = k\} \\
&= \sum_{n=0}^{\infty} \binom{k}{m} \alpha^m (1-\alpha)^n \times \frac{e^{-\lambda t}(\lambda t)^k}{k!} \\
&= \sum_{n=0}^{\infty} \frac{k!}{m!n!} \alpha^m (1-\alpha)^n \times \frac{e^{-\lambda t}(\lambda t)^m (\lambda t)^n}{k!} \\
&= \frac{e^{-\alpha\lambda t}(\alpha\lambda t)^m}{m!} \times \sum_{n=0}^{\infty} \frac{e^{-(1-\alpha)\lambda t}((1-\alpha)\lambda t)^n}{n!} \\
&= \frac{e^{-\alpha\lambda t}(\alpha\lambda t)^m}{m!}.
\end{aligned}$$

Thus, $D(t)$ is a Poisson process with rate $\alpha\lambda$. Similarly, we can show that $R(t)$ is a Poisson process with rate $(1-\alpha)\lambda$.

(ii) To show that the two Poisson processes are independent, it suffices to verify that

$$\begin{aligned}
p\{D(t) = m, R(t) = n\} &= \{D(t) = m, R(t) = n | N(t) = k\} \times p\{N(t) = k\} \\
&= \frac{e^{-\alpha\lambda t}(\alpha\lambda t)^m}{m!} \times \frac{e^{-(1-\alpha)\lambda t}((1-\alpha)\lambda t)^n}{n!} \\
&= p\{D(t) = m\} \times p\{R(t) = n\}. \qquad \square
\end{aligned}$$

Appendix B. Proof of Lemma 2

For some $\widehat{S}_r > 0$, we have

$$\pi(S_s, \widehat{S}_r) = m_d Q_d(S_s, S_r) + m_r Q_r(S_s, S_r) - h_s I_s(S_s, S_r) - h_r I_r(S_s, S_r) \leqslant m_d + m_r - h_s I_s(S_s, \widehat{S}_r) \quad \forall S_s > 0.$$

Therefore, to show that there exists a base-stock level $\overline{S}_s$ such that $\pi(S_s, \widehat{S}_r) \leqslant 0, \forall S_s > \overline{S}_s$, we just need to verify that $\lim_{S_s\to\infty} h_s I_s(S_s, \widehat{S}_r) = \infty$.

Let $T_{S_s} = \sum_{y=0}^{\widehat{S}_r} P_{S_s y}(S_s, \widehat{S}_r)$. To prove $\lim_{S_s\to\infty} h_s I_s(S_s, \widehat{S}_r) = \infty$, it suffices to show $\lim_{S_s\to\infty} h_s S_s^2 T_{S_s} = \infty$ since

$$h_s I_s(S_s, \widehat{S}_r) = h_s \sum_{x=1}^{S_s} \sum_{y=0}^{\widehat{S}_r} x P_{xy}(S_s, \widehat{S}_r) \geqslant h_s S_s^2 \sum_{y=0}^{\widehat{S}_r} P_{S_s y}(S_s, \widehat{S}_r) = h_s S_s^2 T_{S_s}.$$

Since $\lim_{S_s\to\infty} \inf\{T_{S_s} : S_s > 0\} > 0$, there exists N_1 such that $l = \inf\{T_{S_s} : S_s > N_1\} > 0$. Let $L > 0$; since $\lim_{S_s\to\infty} h_s S_s^2 = \infty$, there exists N_2 such that $\forall S_s > N_2$ implies $h_s S_s^2 > \frac{L}{l}$. Now let $N = \max\{N_1, N_2\}$; then, $S_s > N$ implies $h_s S_s^2 T_{S_s} > l\frac{L}{l} = L$. Thus, $\lim_{S_s\to\infty} h_s S_s^2 T_{S_s} = \infty$.

Similarly, we can prove that for some $\widehat{S}_s \geqslant 0$, there exists a base-stock level $\overline{S}_r$ such that $\pi(\widehat{S}_s, S_r) \leqslant 0, \ \forall S_r > \overline{S}_r$. $\square$

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